Swaptions Estimator | fairmat
Version 1.0

1 Introduction

This plug-in allow you to calibrate the two factors Hull and White, and the Pelsser squared gaussian model to a series of observed swaptions prices. The plug-in does not use analytic approximations, but uses Monte Carlo simulation to evaluate the swaptions using one of the available models and fitting them to market data. This strategy permits to the plug-in to operate in a cross-model fashion allowing to be used also to calibrate other models.

2 How to use the plug-in

In the Fairmat user interface when you create a new stochastic process you will find the additional option

- "H&W two factors (community model)". The models are defined by the parameters belonging to the following table.

Description	**Fairmat notation**	**Documentation notation**
	alpha1	α_1
	sigma1	σ_1
	alpha2	α_2
	sigma2	σ_2
	rho	ρ

- "Pelsser Squared Gaussian model". The stochastic process is defined by the parameters shown in table below.

Description	**Fairmat notation**	**Documentation notation**
mean reversion rate (scalar)	a1	a
diffusion parameter (scalar)	sigma1	σ
market price of risk (scalar)	lambda0	$\lambda(t, \omega)$

- In any payoff expression the functions BOND, RATE and CMS (see Fairmat documentation) will be calculated according to the specific model.

3 Swaptions Valuation

For general references on the Swaptions valuation see [1]. Consider a swaption (Swap Option) where we have the right to pay a rate s_K and receive LIBOR on a swap that will last n years starting in T years. We suppose that there are m payments per year under the swap and that the notional is L. Suppose that the swap rate for an n-year swap at the maturity of the swaption is s_T. So the payoff from the swaption consists of mn cash flows equal to

$$\frac{L}{m}\max\left(s_T - s_K\,,\,0\right).$$

If we suppose that these cash flows are received at the payment dates $T_1, T_2, \ldots, T_{mn}$ (approximately $T_i = T + i/m$), each cash flow is the payoff from a call option on s_T with strike price s_K. So the current value of the cash flow received at T_i is

$$\frac{L}{m}P(0,T_i)\left[s_0 N(d_1) - s_K N(d_2)\right] ,$$

where $N(\cdot)$ is the standard normal c.d.f., s_0 is the forward swap rate

$$s_0 = \frac{P(0,T) - P(0,T_{mn})}{\frac{1}{m}\sum_{i=1}^{mn} P(0,T_i)}$$

and

$$\begin{aligned} d_1 &= \frac{\ln\left(s_0/s_K\right) + \sigma^2 T/2}{\sigma\sqrt{T}} \\ d_2 &= d_1 - \sigma\sqrt{T}\,. \end{aligned}$$

The total current value of the swaption is

$$\sum_{i=1}^{mn}\frac{L}{m}P(0,T_i)\left[s_0 N(d_1) - s_K N(d_2)\right] .$$

Defining $A = \frac{1}{m}\sum_{i=1}^{mn} P(0,T_i)$ the value of the swaption becomes

$$LA\left[s_0 N(d_1) - s_K N(d_2)\right] .$$

4 Swaptions Calibration

The estimator tries to minimize the differences between the Black-swaptions prices and the prices of swaptions using one of the available models (for example two factors Hull and White model or Pelsser squared gaussian model), i.e. the objective function is the following

$$\sum_{i=1}^{n}\left(Model^i(\theta) - Black^i\right)^2 , \tag{1}$$

where $Model^i(\theta)$ is the price of the i^{th}-swaption by one of the available models with parameter θ, $Black^i$ the price of the i^{th} swaption by the Black model, and n the number of all swaptions into the Swaptions-Volatility matrix.

4.1 Data for calibration

The swaptions based calibration uses only some specific fields of the file "Interest rate market data xml". In particular:

Market: a string describing the market to which data refer. Possible choices are

- **EU** for Europe
- **US** for USA
- **UK** for United Kingdom
- **SW** for Switzerland
- **GB** for United Kingdom
- **JP** for Japan

Date: is the date to which data refer, the format is ddmmyyyy

ZRMarket: vector with zero coupon rates (continuously compounded rates)

ZRMarketDates: vector of maturities corresponding to ZRMarket

SwaptionTenor: is the year fraction between swap-payments of the swaption

OptionMaturity: vector of Option maturities (i.e. the rows of the following SwaptionsVolatility matrix)

SwapDuration: vector of swap duration (i.e. the columns of the following SwaptionsVolatility matrix)

SwaptionsVolatility: matrix of swaptions-volatilities, i.e. the element SwaptionsVolatility$[i, j]$ is the Black model volatility of a swaption with maturity equal to OptionMaturity$[i]$ and swap duration to SwapDuration$[j]$

An example of an "Interest rate market data xml" containing only these fields is the following


```
<InterestRateMarketData>
<Market>EU</Market>
<Date>31122010</Date>
<ZRMarket>0.012 0.013 0.015 0.019 0.021 0.023 0.024 0.026 0.027 0.029 0.026</ZRMarket>
<ZRMarketDates>1 2 3 5 6 7 8 9 10 30 50</ZRMarketDates>
<SwaptionTenor>1</SwaptionTenor>
<OptionMaturity>0.08 0.25 0.5 1 2 3 4 5 7 10</OptionMaturity>
<SwapDuration>1 2 3 4 5 6 7 9 10 15 20</SwapDuration>
<SwaptionsVolatility>0.432 0.381 0.370 0.361 0.353 0.348 0.346 0.351 0.358 0.366 0.342
0.370 0.379 0.383 0.381 0.379 0.363 0.352 0.348 0.347 0.347 0.326
0.420 0.420 0.408 0.391 0.378 0.360 0.347 0.337 0.331 0.326 0.308
0.480 0.436 0.405 0.380 0.360 0.344 0.332 0.323 0.316 0.311 0.295
```

0.471 0.398 0.362 0.337 0.322 0.310 0.300 0.295 0.290 0.287 0.275
0.418 0.343 0.318 0.300 0.287 0.281 0.276 0.272 0.269 0.266 0.257
0.336 0.290 0.277 0.267 0.259 0.254 0.251 0.248 0.247 0.247 0.241
0.283 0.256 0.248 0.243 0.238 0.234 0.232 0.231 0.231 0.232 0.228
0.229 0.217 0.214 0.211 0.209 0.208 0.208 0.209 0.211 0.212 0.212
0.195 0.187 0.188 0.190 0.192 0.193 0.194 0.196 0.198 0.201 0.205</SwaptionsVolatility>
</InterestRateMarketData>

References

[1] John C. Hull. *Options, futures, and other derivatives.* Prentice Hall, 5th ed. edition, 2002.