Convenience Yield Calculator Version 1.0

1 Introduction

This plug-in implements the capability of calculating instantaneous forward price for commodities like Natural Gas, Fuel Oil and Gasoil. The deterministic forward price model is calibrated starting from an input matrix of futures prices and other market data and specifications.

2 How to use the plug-in

In the Fairmat user interface under the parameter and symbols you'll find, under the category "Transformations", the Convenience Yield Calculator which provides a way to calculate, as a function of several input, the convenience yield of Natural Gas, Fuel Oil and Gasoil.

Convenience Yield Calculator is defined by the following input parameters which may be also expressed as other Fairmat project parameters:

- Futures Matrix: a matrix that contains the start dates, end dates and relative futures prices of commodity;

- Zero Rate Curve: an interpolated function that contains dates and level of risk-free curve defined in the same currency of futures prices;

- Working days: the number of days to be considered for calculation of dates in year fraction;

- Spot price: the constant that identifies the spot price of the modelled commodity.

Figure 1 shows an input form which specifies a convenience yield function from the input listed above. The resulting convenience yield can be used in Fairmat expressions as a user defined function. The plug-in also defines functions which take a convenience yield function as one of their arguments:

The function `CommodityForward` calculates the forward prices at *Valuation Date* (t_0). It is expressed in Fairmat language as follows:
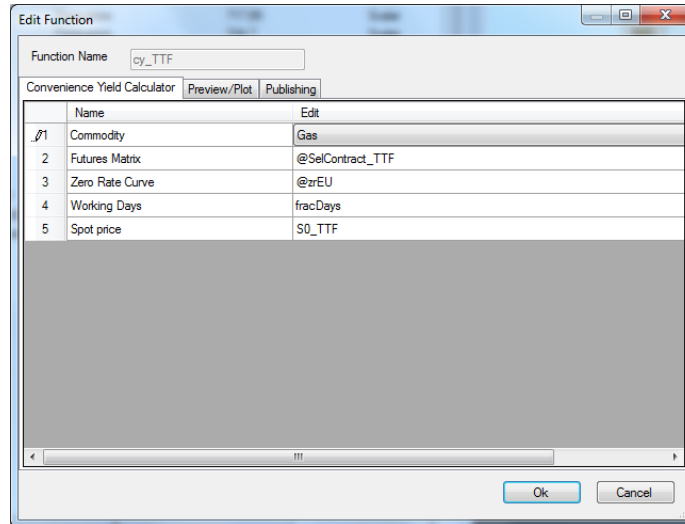
- `CommodityForward(@CV;@zr;T);`

Figure 1: *Example: convenience yield calculator input form.*

- `CommodityForward(@CV;@zr;T;@HistPrice);`

where:

- `CV` is the convenience yield of underlying commodity, defined using the "Convenience Yield Calculator";

- `zr` is the zero rate curve, which must be defined in the same currency of the commodity (it is need to calculate the discount factor);

- `T` is the delivery date;

- `HistPrice` is a function representing the commodity historical prices time series.

Furthermore the function `CommodityForwardIst` which is used to calculate a instantaneous forward with, continuous delivery, into a delivery period $[T_s, T_e]$ at a defined date t. It is expressed in Fairmat language as follows:
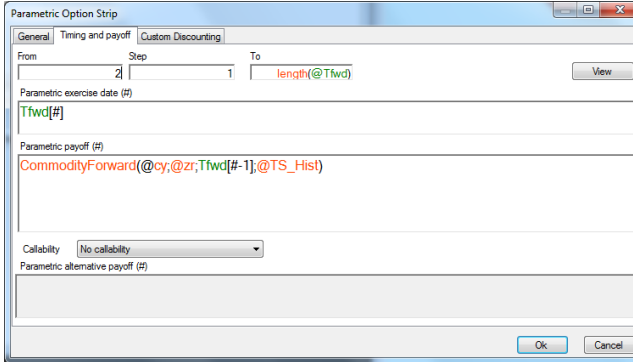
- `CommodityForwardIst(@CV;@zr;t;Ts;Te);`

- `CommodityForwardIst(@CV;@zr;t;Ts;Te;@HistPrice);`

where:

- `CV` is the convenience yield of the underlying commodity defined using the "Convenience Yield Calculator";

- `zr` is the zero rate curve, which must be defined in the same currency of the commodity (it is need to calculate the discount factor);
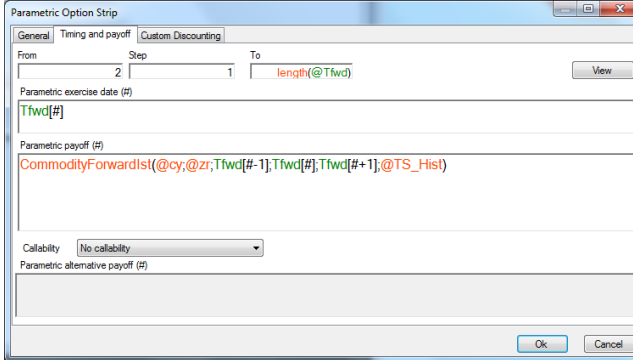
- **t** is the starting date used to calculate the instantaneous forward and refers to the delivery period $[T_s, T_e]$;

- **Ts** is the start date where begins the delivery period;

- **Te** is the end date that defined the end of delivery period;

- **HistPrice** is a function representing the commodity historical prices time series.

As it can be noticed in Figure 2, these two functions can be used into option map expressions either in a single payment leg or in a sequence of payments (option strip block).

(a) CommodityForward function

(b) CommodityForwardIst function

Figure 2: *Example: use of* `CommodityForward` *and* `CommodityForwardIst` *in an option map block.*

3 Fairmat Example

As Case Study, we have implemented a simple contract based on the weighting of Fuel Oil and Natural Gas price averages. In particular, the price of this contract is the weighted sum of the average of Fuel Oil monthly price, reset into previous 9 months and of the average price of daily spot price of Natural Gas reset into previous 3 months plus a Spread. The delivery period is settled every 3 months and the contract maturity is 3 years.

The Fairmat example for the contract can be downloaded from the following link http://www.fairmat.com/resources/view/convenience-yield-calculator-example.

4 Implementation Details

4.1 Bootstrapping of non–overlapping forwards

We denote $F(t, T_s^j, T_e^j)$ the forward price at t of contract j for delivery of the commodity uniformly from the starting date T_s^j to the ending date T_e^j at a rate of one unit per day, for $j = 1, \ldots, J$.

In case of which the settlement periods $[T_s^j, T_e^j]$ of the different contracts traded on a given commodity are overlapping, by bootstrapping on the existing traded contracts, we determine the prices of forward contracts on non–overlapping time intervals.

For example, given two contracts $F(t_0, T_s^1, T_e^1)$ and $F(t_0, T_s^2, T_e^2)$ such that $T_s^1 < T_s^2 < T_e^1 < T_e^2$. In this case we define a vector of dates

$$(t_0, t_1, t_2, t_3, t_4) = (t_0, T_s^1, T_s^2, T_e^1, T_e^2)$$

and using bootstrap with come up with a set of forward prices $\{F(t_0, t_i, t_{i+1}), i = 1, 2, 3\}$ whose settlement periods are not overlapping, and satisfy the following no–arbitrage conditions

$$F(t_0, T_s^1, T_e^1) = \frac{t_2 - t_1}{t_3 - t_1} F(t_0, t_1, t_2) + \frac{t_3 - t_2}{t_3 - t_1} F(t_0, t_2, t_3)$$

$$F(t_0, T_s^2, T_e^2) = \frac{t_3 - t_2}{t_4 - t_2} F(t_0, t_2, t_3) + \frac{t_4 - t_3}{t_4 - t_2} F(t_0, t_3, t_4).$$

By extending the same idea to all J forward contracts, a vector of dates $t_0, t_1, \ldots, t_n$ and the set of prices

$$\{F(t_0, t_i, t_{i+1}), i = 1, \ldots, n-1\}$$

The main is coverage all periods from the first start date to the last end date of future prices.

4.2   Defining the model

Given forward price for delivery at date s as

$$f(t,s) = S_t \frac{e^{-y(t,T_1,T_2)(s-t)}}{B(t,s)}, \tag{1}$$

where

- $S_t = f(t,t)$ is the spot price;

- $B(t,s)$ is the price at t of a zero–coupon bond maturing at s;

- $y(t,T_1,T_2)$ is the net convenience yield (i.e., the convenience yield net of the storage cost) implicit in the quoted forward price at t for delivery in the interval $[T_1,T_2]$.

Given the futures prices with uniform delivery within an interval, the plug-in derive the forward prices for delivery in a specific date or sub–period (e.g., a week), as detailed here below. Dividing the settlement interval, say $[T_1,T_2]$, into a integer number of days (or weeks, or other equal sub–periods) under no–arbitrage hypothesis,

$$F(t,T_1,T_2) = \frac{1}{A(t,T_1,T_2)} \sum_{s=T_1}^{T_2} B(t,s)f(t,s), \tag{2}$$

where

$$A(t,T_1,T_2) = \sum_{s=T_1}^{T_2} B(t,s)$$

is the value of a constant annuity paying \$1 a day (or sub-period) from T_1 to T_2, and $f(t,s)$ is the forward price at t for one unit of the commodity delivered at s.

Using this definition of the forward price into equation (2), we can related the quoted forward prices to convenience yields:

$$F(t,T_1,T_2) = S_t \frac{1}{A(t,T_1,T_2)} \sum_{s=T_1}^{T_2} e^{-y(t,T_1,T_2)(s-t)}.$$

The "Convenience Yield Calculator" resolve this equation it numerically for $y(t,T_1,T_2)$.

4.3   Instantaneous Forward Calculation

The plug-in provide to calculate the instantaneous forward of a commodity with a continuous delivery.

Assuming that forward prices are martingales under the risk–neutral probability as

$$\mathbb{E}_0^* [f(t,s)] = f(t_0,s),$$

where $f(t_0, s)$ can be calculated using the convenience yields in (**??**) and equation (1) for $t = t_0$.

Assuming no correlation between the commodity price and the interest rates, we have

$$\mathbb{E}_0^* \left[F(t, T_i, T_{i+1}) \right] = \frac{1}{A_0(t, T_i, T_{i+1})} \sum_{s=T_i}^{T_{i+1}} B_0(t, s) \mathbb{E}_0^* \left[f(t, s) \right]$$

$$= \frac{1}{A_0(t, T_i, T_{i+1})} \sum_{s=T_i}^{T_{i+1}} B_0(t, s) f(t_0, s),$$

where $B_0(t, s) = B(t_0, s)/B(t_0, t)$ is the price of a zero coupon bond at t with maturity in s, calculated using the forward zero rates implicit in the zero rate curve observed at t_0, and

$$A_0(t, T_i, T_{i+1}) = \sum_{s=T_i}^{T_{i+1}} B_0(t, s)$$

is the annuity calculated using such zero coupon prices, for s ranging from T_i to T_{i+1}.