Tutorial | fairmat

Interest Rate Swaps Modelling

This tutorial shows how Fairmat ***Academic*** can be used for *Swap modelling*, as found in *John C. Hull "Options, futures and other derivatives" [Chapter 6, 5th Edition]*.

In particular:

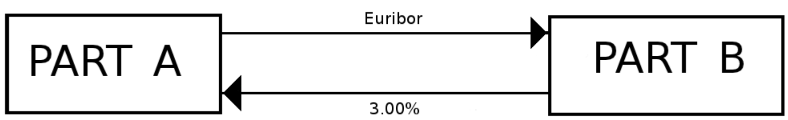
- Calculate the convenience of an IRS
- To handle day conventions
- To calculate the expected value of a SWAP

You can also find a video for this tutorial at: http://youtu.be/Zib0joUXY94

1 Interest Rate Swaps

An *Interest Rate Swap* (IRS) involves two parties exchanging fixed for variable interest rates on a national amount over a series of payment dates.

Consider for example a three-year IRS between PART A and PART B , the starting day is *t=07/04/2008*. We assume that PART B agrees to pay PART A an interest rate of 3% for annum (semiannual compounding), and PART A agrees to pay PART B the six-month *Euribor* rate. For both parties, the notional amount is $100.



Payment dates and observed rates are summarized in the following table:

Date	**Euribor 6 month**	**Fix 6 month**
07/04/2008	*Contract initial date*	*Contract initial date*
07/10/2008	2,90%	3,00%
07/04/2009	3,50%	3,00%
07/10/2009	3,20%	3,00%
07/04/2010	2,60%	3,00%
07/10/2010	3,10%	3,00%
07/04/2011	3,70%	3,00%

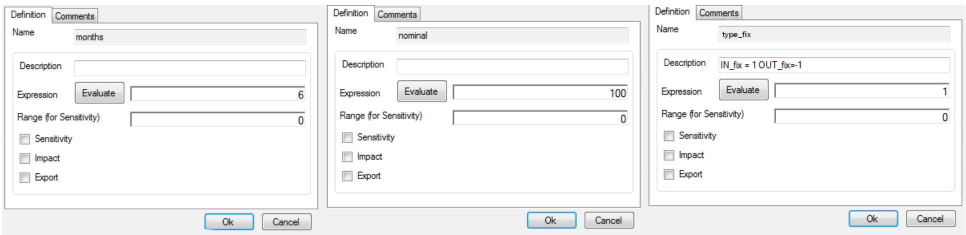
The following questions may arise:

- What are the fixed and variable cash flows that the two parties must pay?
- Is this *SWAP* profitable for *PART_A*?

With Fairmat it is easy to answer to these questions.

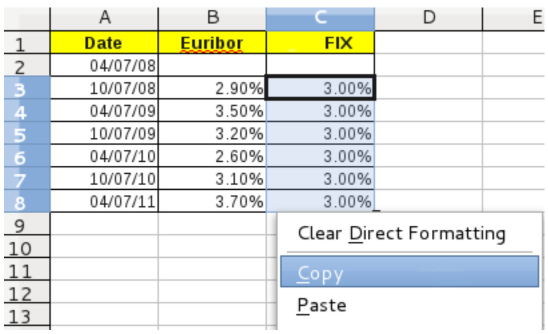
The following data are available:

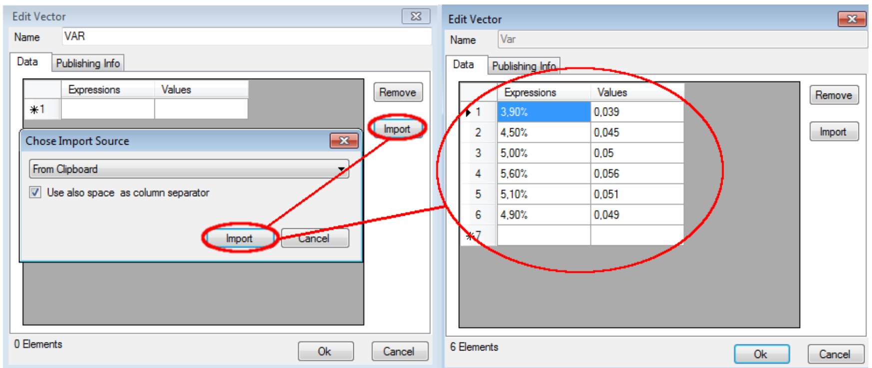
- *notional*: notional amount $100;
- *months*: payment's period with month with 6 semiannual compounding;
- *type_fix*: 1 long position with fix-rate, (-1) long position with floating-rate;
- *FIX*: vector containing the fixed rate of each payment date;
- *Var*: vector containing the floating rate of each payment date;



`Parameters:` create *nominal*, *months* and *type_fix* with `Constant Parameter` (`Parameters & Functions` → `Add` → `Constant Parameter`).

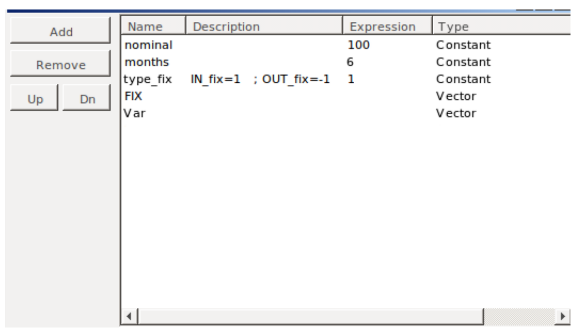
FIX and *VAR* are vectors, so you may import them from a spreadsheet. To do it select and copy the relevant column





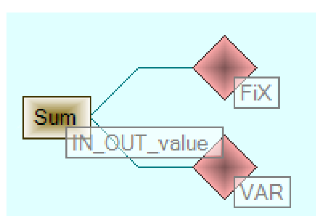
Create a vector: select `Parameters & Functions` → `Vector of value/expressions/date` and `import data`.

Our model's parameters are show below



Name	Description	Expression	Type
nominal		100	Constant
months		6	Constant
type_fix	IN_fix=1 ; OUT_fix=-1	1	Constant
FIX			Vector
Var			Vector

In order to calculate the payments, open the `Option Map` and create the Custom Option (the pink rhombus):



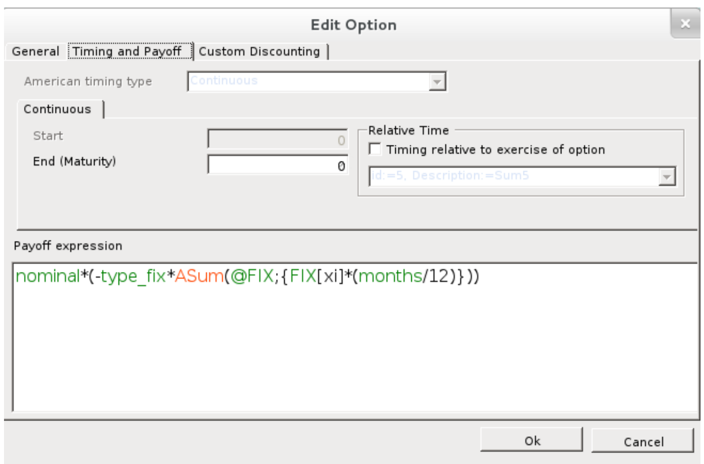
In this case the option map blocks are

- Blocks *FIX* and *VAR* calculate the total payment of the fix/variable rates about nominal amount

$$nominal * (-type\_fix * ASum(@FIX; \{FIX[xi] * (month/12)\}))$$

$$nominal * (type\_fix * ASum(@VAR; \{VAR[xi] * (month/12)\}))$$

`ASum`[1] is an operator that calculates the sum of the evaluation of the expression indicated, where ***xi*** refers to the value of the current element of the vector @FIX/@VAR.[2]

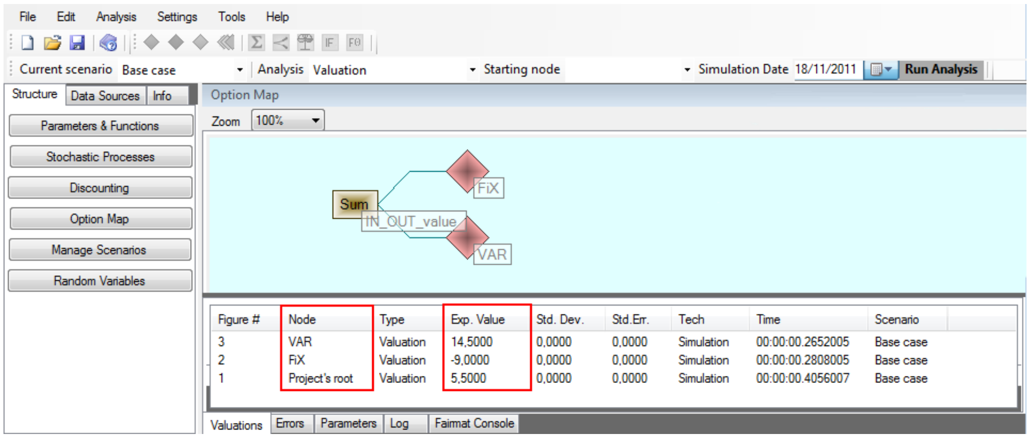


- Block *IN_OUT_value*: it sums the value for *FIX* and *VAR* legs.

Sum Operator can omit the need for writing the blocks in line (see Tutorial #1 page 2 (Method 2)).

[1] For more information consult `Help` in Fairmat ***Academic***.
[2] For write the symbol `@` before vector name.

Choose a *Starting Node* and click on `Run Analysis`.

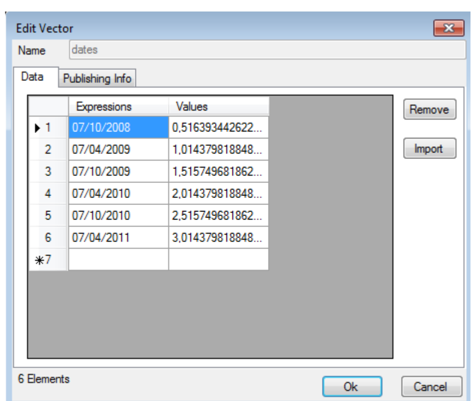


Report here what we get from the program that is helpful to answer the question.

2 IRS and Day count conventions

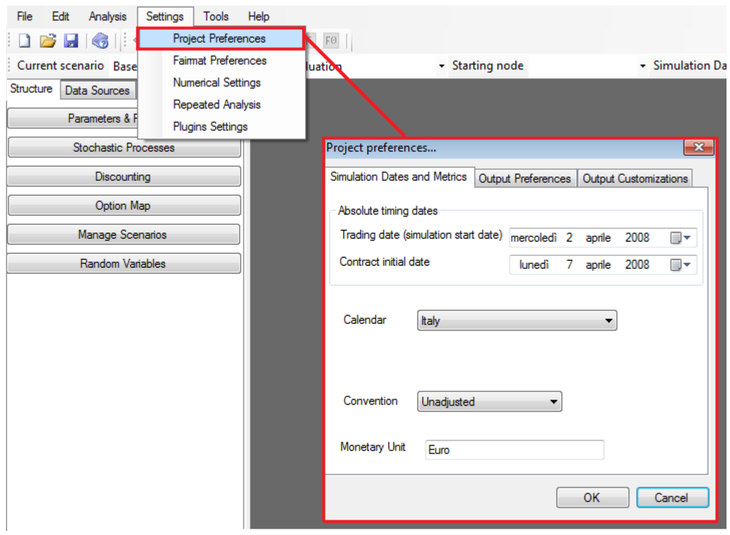
For simplicity we ignored day count issues. In reality they must be properly considered. With Fairmat is easy to take them into account.

Consider the case presented in *Section 1* and recreate the variable *nominal, type_fix, FIX* and *VAR* and create a vector of payment dates. In the figure below, the column *Value* depends on the `Simulation Date` and represents the difference, in fractions of year, between the date indicated in the row and the simulation date



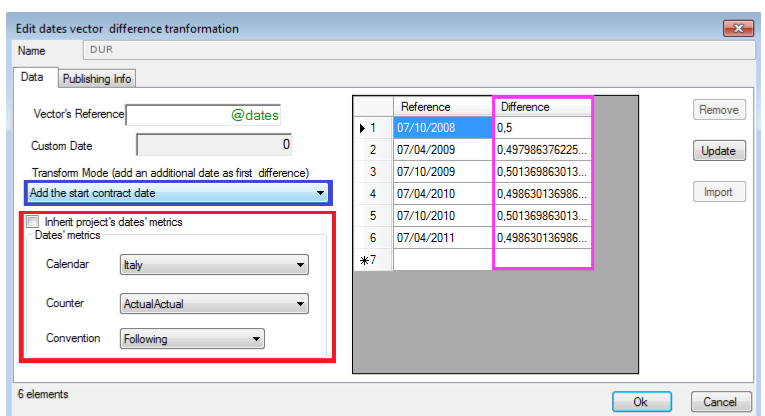
Note: insert only the payment dates.

In Fairmat you can indicate the contract starting date, in this case *07/04/2008* (note: this isn't a payment date).



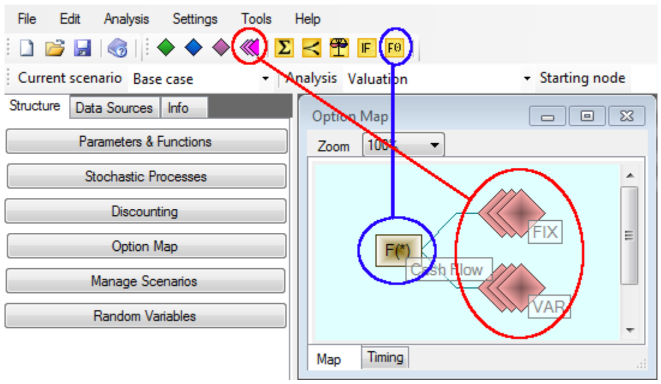
Click on `Setting` → `Project Preferences`. In the window indicate initial *contract date* and a Trading date (*simulation start date*).

Create a new symbol: `Date vector difference calculator` (*DUR*). Indicate a Vector's Reference (`@`*dates*) and a *Custom Date*, which is the initial date in Project Preferences.



The combo box (in blue) indicates the date on which to start the transformation, the red indicates the convention of adjusting dates specified or determined with respect to a transaction, and the pink indicates the difference between the actual date and the previous one.

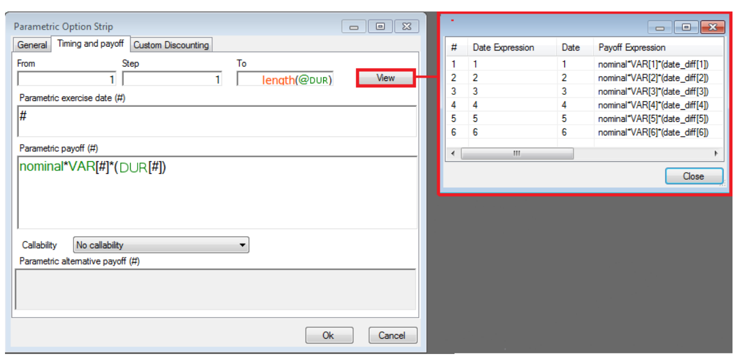
To enter the cash flows, open the `Option Map` and create a `strip of options` (pink rhombus), which can handle a sequences of payments.



`Strip of Options and Functional Operator`

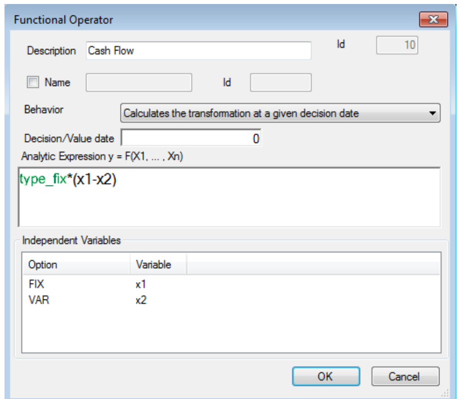
Here the blocks are:

- *FIX* and *VAR* legs calculate the total payment of the fix/floating rates about nominal amount. Options Strips simplifies the repetition of similar payoffs and exercise dates (by allowing to parametrize expressions using the character #), and summing them over the components of an input vector.



`Parametric Options Strip`: the payoff *nominal*VAR[#]*(DUR[#])* is calculated for element in Vector in a determinate position expressed that takes the values 1,2,...,**length(**@DUR**)**

- The block *functional operator* calculates the cash flow of long or short fixed positions.



Note: at the bottom of the function operator window there are variables `x1` and `x2`. These indicates the values of subsequent nodes in the option map, respectively *FIX* and *VAR*.

Choose a *Starting Node* and click on `Run Analysis` to obtain the output:

Figure #	Node	Type	Exp. Value	Std. Dev.	Std.Err.	Tech	Time	Scenario
3	Project's root	Valuation	5,4964	0,0000	0,0000	Simulation	00:00:00.3588007	Base case
2	VAR	Valuation	14,4904	0,0000	0,0000	Simulation	00:00:00.2496004	Base case
1	FIX	Valuation	8,9940	0,0000	0,0000	Simulation	00:00:00.3432006	Base case

Valuations | Errors | Parameters | Log | Fairmat Console

3 Describe how the output is useful for the specific problem you are describing

To calculate the value of a *SWAP*, that is value of FIX and VAR legs at a date following the initial date, you have to compute, the following expression:

$$V\_FIX = K \cdot e^{-rt_1} + K \cdot e^{-rt_2} + ... + (K + Q) \cdot e^{-rt_n} \tag{1}$$

$$V\_VAR = (K^* + Q) \cdot e^{-rt_n} \tag{2}$$

where

$$K = \frac{(S_n \cdot Q)}{n}$$

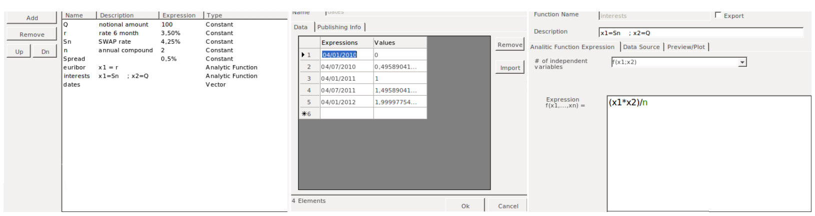
$$K^* = \frac{((euribor + spread) \cdot Q)}{n} = \frac{((n(e^{r/n} - 1) + spread) \cdot Q)}{n}$$

IN the equations:

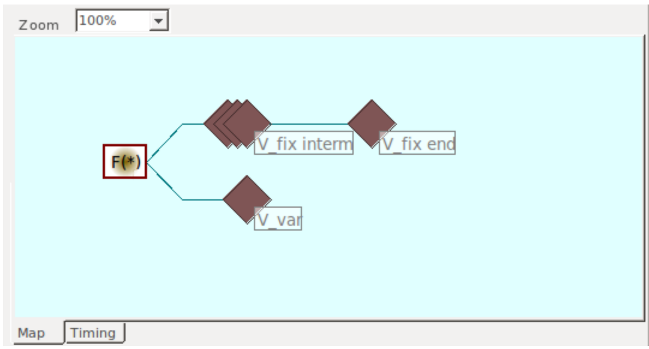
- n is the annual compound frequency
- r is the fix rate
- S_n is the Swap rate
- Qis the notional amount

In Fairmat we can solve it as follows:

- Create the following constants:
  - Q, r, Sn, n, $spread$;
  - $euribor$: the euribor formula $(n(e^{r/n} - 1))$;
  - $interests$: used for calculate the value of K and K^*;
  - $dates$: the vector of payment dates;

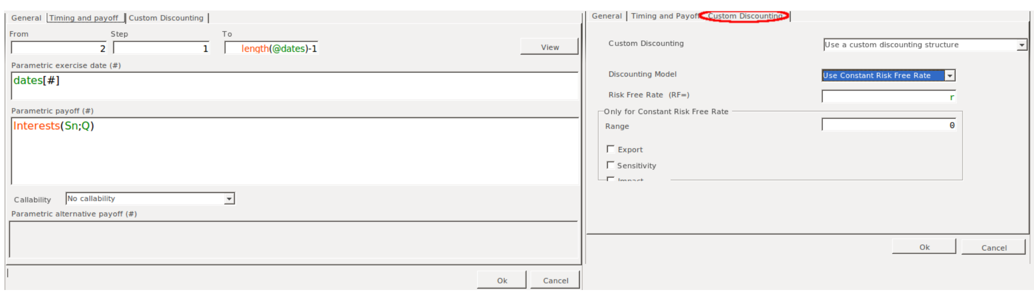


- Create the contract structure using the Option map



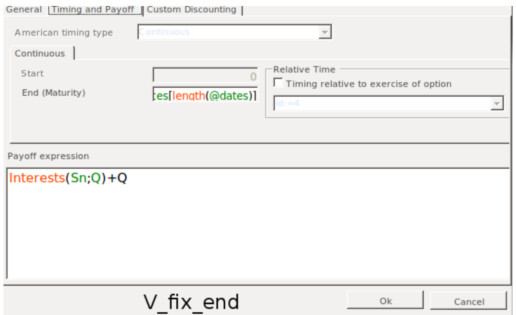
The option map blocks are

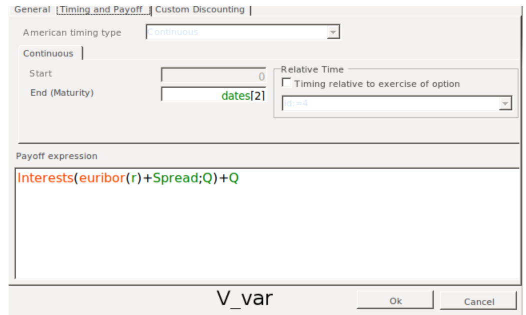
- *V_fix_interm* which calculates the present value of the interest payments of the fix rate, excluding the end date payment



Note: Insert the formula (1) and set discounting in `Custom Discounting`. The payoff is calculated for every payment date defined by the expression *dates(#)*, where `#` takes the values 1,2,...,(`length(@dates)-1`). In the discount settings it possible to choose from the following discount models (see Tutorial #1 page 5). We are interested in the `Risk free Rate`.

- *V_fix_end* which calculates the final payment for the fixed leg $(K+Q) \cdot e^{-rt_n}$
- Block *V_var* which calculates the payment for the floa





The expression above represent the formulas (1) and (2). In order to discount them you must insert at the `End (Maturity)` the final payment, then click on `Custom Discounting` to insert the discount rate.

- Block *operator F(*)*: it is the evaluation of swap, in each case (*V_var*-*V_fix*);

To do the calculation choose the `Simulation date` and click on `Run Analysis`. After that you will see the valuation result in the bottom panel (Valuation tab). You can also calculate the value for each node, or change the valuation/Simulation date.