

Interest Rate Swaps Modelling

This tutorial shows how Fairmat **Academic** can be used for Swap modelling, as found in John C. Hull "Options, futures and other derivatives" [Chapter 6, 5th Edition].

In particular:

- Calculate the convenience of an IRS
- To handle day conventions
- To calculate the expected value of a SWAP

You can also find a video for this tutorial at: http://youtu.be/Zib0joUXY94



1 Interest Rate Swaps

An *Interest Rate Swap* (IRS) involves two parties exchanging fixed for variable interest rates on a national amount over a series of payment dates.

Consider for example a three-year IRS between PART A and PART B , the starting day is $t{=}07/04/2008$. We assume that PART B agrees to pay PART A an interest rate of 3% for annum (semiannual compounding), and PART A agrees to pay PART B the six-month *Euribor* rate. For both parties, the notional amount is \$100.



Payment dates and observed rates are summarized in the following table:

Date	Euribor 6 month	Fix 6 month
07/04/2008	Contract initial date	Contract initial date
07/10/2008	$2,\!90\%$	$3{,}00\%$
07/04/2009	$3{,}50\%$	$3{,}00\%$
07/10/2009	$3{,}20\%$	$3{,}00\%$
07/04/2010	$2{,}60\%$	$3{,}00\%$
07/10/2010	$3,\!10\%$	$3{,}00\%$
07/04/2011	3,70%	$3{,}00\%$

The following questions may arise:

- What are the fixed and variable cash flows that the two parties must pay?
- Is this SWAP profitable for $PART_A$?

With Fairmat it is easy to answer to these questions.



The following data are available:

- notional: notional amount \$100;
- months: payment's period with month with 6 semiannual compounding;
- type_fix: 1 long position with fix-rate, (-1) long position with floating-rate;
- *FIX*: vector containing the fixed rate of each payment date;
- Var: vector containing the floating rate of each payment date;

Definition Comments	Definition Comments	Definition Comments
Name months	Name nominal	Name type_fix
Description	Description	Description IN_fix = 1 OUT_fix=-1
Expression Evaluate 6	Expression Evaluate 100	Expression Evaluate 1
Range (for Sensitivity) 0	Range (for Sensitivity) 0	Range (for Sensitivity) 0
Sensitivity	Sensitivity	Sensitivity
mpact	Impact	Impact
Export	Export	Export
Ok Cancel	Ok Cancel	Ok Cancel

Parameters: create *nominal*, *months* and *type_fix* with Constant Parameter (Parameters & Functions \rightarrow Add \rightarrow Constant Parameter).

FIX and $V\!AR$ are vectors, so you may import them from a spread sheet. To do it select and copy the relevant column

	A	В	С	D	E
1	Date	Euribor	FIX		
2	04/07/08				
3	10/07/08	2.90%	3.00%		
4	04/07/09	3.50%	3.00%		
5	10/07/09	3.20%	3.00%		
6	04/07/10	2.60%	3.00%		
7	10/07/10	3.10%	3.00%		
8	04/07/11	3.70%	3.00%		
9			Clear Dir	ect Formatt	ina
10				eccronnacc	ing
11			<u>С</u> ору		
12			Basta		
13			raste		





Create a vector: select Parameters & Functions \rightarrow Vector of value/expressions/date and import data.

Our model's parameters are show below

1	Namo	Description	Exprossion	Tupo
Add	nominal	Description	100	Constant
Remove	months		6	Constant
	type_fix	IN_fix=1 ; OUT_fix=-1	1	Constant
Up Dn	FIX			Vector
	Var			Vector
	•			Þ

In order to calculate the payments, open the Option Map and create the Custom Option (the pink rhombus):





In this case the option map blocks are

• Blocks FIX and VAR calculate the total payment of the fix/variable rates about nominal amount

```
nominal * (-type_fix * ASum(@FIX; {FIX[xi] * (month/12)}))
nominal * (type_fix * ASum(@VAR; {VAR[xi] * (month/12)}))
```

 $ASum^1$ is an operator that calculates the sum of the evaluation of the expression indicated, where *xi* refers to the value of the current element of the vector $@FIX/@VAR.^2$

	~
General Timing and Payoff Custom Discounting	
American timing type Continuous	
Continuous	
Start 0 Relative Time	
End (Maturity) 0	
Ju.=3, Description.=sum5	_
Payoff expression	
nominal*(-type_fix*ASum(@FIX;{FIX[xi]*(months/12)}))	
Ok Cance	1

• Block IN_OUT_value: it sums the value for FIX and VAR legs.

Sum Operator can omit the need for writing the blocks in line (see Tutorial #1 page 2 (Method 2)).



 $^{^1 {\}rm For}$ more information consult Help in Fairmat A cademic.

 $^{^2 {\}rm For}$ write the symbol @ before vector name.

Choose a Starting Node and click on Run Analysis.

File Edit Analysis Settings	Tools He	lp								
🗋 💕 🖬 🌏 🌢 🔶 🌾	\ge	< 👚 IF F0								
Current scenario Base case	+ Ana	lysis Valuation		 Starting 	node		 Simulati 	on Date 18/11/2011	Run Anal	ysis
Structure Data Sources Info	Option Map									
Parameters & Functions	Zoom 100	" ▼								
Stochastic Processes										
Discounting		_		FiX						
Option Map		Sum	K_OUT_val	ue						
Manage Scenarios				VAR						
Random Variables										
	Figure #	Node	Туре	Exp. Value	Std. Dev.	Std.Err.	Tech	Time	Scenario	
	3	VAR	Valuation	14,5000	0,0000	0,0000	Simulation	00:00:00.2652005	Base case	
	1	FIX Project's root	Valuation Valuation	-9,0000 5,5000	0,0000	0,0000	Simulation	00:00:00.2808005	Base case Base case	
		· ·	1	L	1					
	Valuations	Errors Parameter	s Log F	airmat Console						

Report here what we get from the program that is helpful to answer the question.

2 IRS and Day count conventions

For simplicity we ignored day count issues. In reality they must be properly considered. With Fairmat is easy to take them into account.

Consider the case presented in Section 1 and recreate the variable nominal, $type_fix$, FIX and VAR and create a vector of payment dates. In the figure below, the column Value depends on the Simulation Date and represents the difference, in fractions of year, between the date indicated in the row and the simulation date

Ed	it Vec	tor		X
Na	ame	dates		
D	ata	Publishing Info		
1		Expressions	Values	Remove
	▶1	07/10/2008	0,516393442622	
	2	07/04/2009	1,014379818848	Import
	3	07/10/2009	1,515749681862	
	4	07/04/2010	2,014379818848	
	5	07/10/2010	2,515749681862	
	6	07/04/2011	3,014379818848	
	*7			
ļ				
6	Elemen	ts		Ok Cancel

Note: insert only the payment dates.





In Fairmat you can indicate the contract starting date, in this case 07/04/2008 (note: this isn't a payment date).

Click on Setting \rightarrow Project Preferences. In the window indicate initial contract date and a Trading date (simulation start date).

Create a new symbol: Date vector difference calculator (DUR). Indicate a Vector's Reference (@dates) and a *Custom Date*, which is the initial date in Project Preferences.

Edit dates vector difference tranformation					×
Name					
Data Publishing Info					
Vector's Beference		Reference	Difference		Remove
Guales	▶1	07/10/2008	0,5		
Custom Date 0	2	07/04/2009	0,497986376225		Update
Transform Mode (add an additional date as first difference)	3	07/10/2009	0,501369863013		
Add the start contract date	4	07/04/2010	0,498630136986		Import
Inherit project's dates' metrics	5	07/10/2010	0,501369863013		
- Dates' metrics	6	07/04/2011	0,498630136986		
Calendar Italy 🕶	*7				
Counter ActualActual 🗸					
Convention Following -					
6 elements				Ok	Cancel

The combo box (in blue) indicates the date on which to start the transformation, the red indicates the convention of adjusting dates specified or determined with respect to a transaction, and the pink indicates the difference between the actual date and the previous one.





To enter the cash flows, open the $\tt Option\ Map$ and create a $\tt strip\ of\ options\ (pink\ rhombus),\ which\ can\ handle\ a\ sequences\ of\ payments.$

Strip of Options and Functional Operator

Here the blocks are:

• *FIX* and *VAR* legs calculate the total payment of the fix/floating rates about nominal amount. Options Strips simplifies the repetition of similar payoffs and exercise dates (by allowing to parametrize expressions using the character #), and summing them over the components of an input vector.

Parametric Option Strip		1			
General Timing and payoff Custom Discounting		#	Date Expression	Date	Payoff Expression
From Step To		1	1	1	nominal*VAR[1]*(date_dff[1])
1 1 length(@pur)	View	2	2	2	nominal*VAR[2]*(date_diff[2])
Parametric exercise date (#)		3	3	3	nominal*VAR[3]*(date_diff[3])
		4	4	4	nominal*VAR[4]*(date_dff[4])
#		5	5	5	nominal*VAR[5]*(date_diff[5])
		6	6	6	nominal*VAR[6]*(date_dff[6])
Parametric payoff (#)		•			F.
					Close
Callability No callability Parametric alternative payoff (#)					
Ok	Cancel				

Parametric Options Strip: the payoff *nominal*VAR[#]*(DUR[#])* is calculated for element in Vector in a determinate position expressed that takes the values 1,2,...,length(@DUR)



• The block *functional operator* calculates the cash flow of long or short fixed positions.

Functional Op	erator								×
Description	Cash F	ow					ld		10
Name				ld					
Behavior		Calculates the	transfor	matior	n at a gi	iven deci	ision date		•
Decision/Valu Analytic Expre	ue date ssion y =	F(X1, , Xn)			0				
type_fix*(x	(1-x2) Variables								
Option		Variable							
FIX VAR		x1 x2							
							ОК	Ca	ancel

Note: at the bottom of the function operator window there are variables x1 and x2. These indicates the values of subsequent nodes in the option map, respectively *FIX* and *VAR*.

Choose a *Starting Node* and click on Run Analysis to obtain the output:

Figure #	Node	Туре	Exp. Value	Std. Dev.	Std.Err.	Tech	Time	Scenario	
3	Project's root	Valuation	5,4964	0,0000	0,0000	Simulation	00:00:00.3588007	Base case	
2	VAR	Valuation	14,4904	0,0000	0,0000	Simulation	00:00:00.2496004	Base case	
1	FIX	Valuation	8,9940	0,0000	0,0000	Simulation	00:00:00.3432006	Base case	
		1							
									_
Valuations	Errors Parameter	ers Log I	aimat Console						

3 Describe how the output is useful for the specific problem you are describing

To calculate the value of a SWAP, that is value of FIX and VAR legs at a date following the initial date, you have to compute, the following expression:

$$V_FIX = K \cdot e^{-rt_1} + K \cdot e^{-rt_2} + \dots + (K+Q) \cdot e^{-rt_n}$$
(1)

$$V_{-}VAR = (K^* + Q) \cdot e^{-rt_n} \tag{2}$$

where



$$K = \frac{(S_n \cdot Q)}{n}$$
$$K^* = \frac{((euribor + spread) \cdot Q)}{n} = \frac{((n(e^{r/n} - 1) + spread) \cdot Q)}{n}$$

IN the equations:

- n is the annual compound frequency
- r is the fix rate
- S_n is the Swap rate
- Qis the notional amount

In Fairmat we can solve it as follows:

- Create the following constants:
 - -Q, r, Sn, n, spread;
 - euribor: the euribor formula $(n(e^{r/n} 1));$
 - *interests*: used for calculate the value of K and K^* ;
 - *dates*: the vector of payment dates;







• Create the contract structure using the Option map

The option map blocks are

• *V_fix_interm* which calculates the present value of the interest payments of the fix rate, excluding the end date payment

General Timing and payoff Custom Discounting		General Timing and Payoff Custom Discounting	
Step To 2 1 length(@dates)-1	View	Custom Discounting	Use a custom discounting structure
Parametric exercise date (#)			
dates[#]		Discounting Model	Use Constant Risk Free Rate 💌
		Risk Free Rate (RF=)	r
Parametric payoff (#)		Only for Constant Risk Free Rate	
Interests(Sn;Q)		Range	0
		E functi	
		T benaduly	
Callability No callability			
Parametric alternative payoff (#)			
			Ok Cancel
	Ok Cancel		

Note: Insert the formula (1) and set discounting in Custom Discounting. The payoff is calculated for every payment date defined by the expression dates(#), where # takes the values 1,2,...,(length(@dates)-1). In the discount settings it possible to choose from the following discount models (see Tutorial #1 page 5). We are interested in the Risk free Rate.



- V_{fix} -end which calculates the final payment for the fixed leg $(K+Q) \cdot e^{-rt_n}$
- Block $V_{-}var$ which calculates the payment for the floa

General Timing and Payoff Custom Discounting	General Timing and Payoff Custom Discounting
American timing type Continuous Start O Felative Time Timing relative to exercise of option	American timing type Continuous Continuous Start 0 Felative Time start 0 Felative Time
End (Maturity) [esflength(@dates)]	Image: Second
Interests(Sn;Q)+Q	Interests(euribor(r)+Spread;Q)+Q
V fix end	el V_var Cancel

The expression above represent the formulas (1) and (2). In order to discount them you must insert at the End (Maturity) the final payment, then click on Custom Discounting to insert the discount rate.

• Block operator F(*): it is the evaluation of swap, in each case (V_var-V_fx) ;

To do the calculation choose the Simulation date and click on Run Analysis. After that you will see the valuation result in the bottom panel (Valuation tab). You can also calculate the value for each node, or change the valuation/Simulation date.

