

Forward Prices & Forward Contracts

In this tutorial we show you how to use *Fairmat Academic* to solve exercises on forward prices modelling, by showing you how to answer questions found in John C. Hull Options, futures and other derivatives [Chapter 3 "Determination of forward and futures prices", 5^{th} Edition].

You can also find a video for this tutorial at: http://youtu.be/Bq2B61cf9FU

<u>@@@@</u>

1 Zero coupon bond

We want to calculate a forward price, at time t^1 of an underlying price that does not make periodic interest payments:

$$F_t = S_t \cdot e^{Rf_t(T-t)}$$

The data available are:

- underlying price (**S**), price in stipulation date;
- Risk Free (**Rf**);
- Maturity $(\mathbf{EX})^2$;

With Fairmat, the first step is creating the problem constants as follows:

- 1. Click in the window Structure the Parameters & Functions;
- 2. Click in the new opened window the Add button ;
- 3. Select Constant Parameter on the list;





 $^{^{1}\,}t$ is indicated in Simulation Dates.

 $^{^{2}}EX$ is T in the formula.

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In order to focus on a specific case, constants can be initialized as follows: $S_t{=}40\$,\,Rf_t{=}5\%,\,{\rm EX{=}01/04/2012}$

| Parameter or Constant Expression | Parameter or Constant Expression | Parameter or Constant Expression |
|----------------------------------|----------------------------------|----------------------------------|
| Definition Comments | Definition Comments Name pr | Definition Comments |
| Description | Description | Description |
| Expression Evaluate 40 | Expression Evaluate 5% | Expression Evaluate 01/04/2012 |
| Range (for Sensitivity) 0 | Range (for Sensitivity) 0 | Range (for Sensitivity) 0 |
| Sensitivity | Sensitivity | Sensitivity |
| Impact | mpact | Impact |
| Export | Export | Export |
| | | |
| Ok Cancel | Ok Cancel | Ok Cancel |

Then we can create the forward expression using Fairmat's analytic functions (just go to Parameters & Functions and select Analytic Function).

In this example we model forward prices as a function of three independent variables, which as indicated $1=S_t \ x2=Rf_t \ x3=Maturity$:

| Edit Function | | |
|-------------------------------|-----------------------------------|--------|
| Function Name | F Export | |
| Description | prezzo foward: f(S,Rf,EX) | |
| Analitic Function Exp | oression Data Source Preview/Plot | |
| # of independent variables | [t/(1x2x3) • | |
| Expression f(x1xn) = | x1*exp(x2*x3) | |
| | | |
| | Ok | Cancel |

In the expression textbox just write the forward formula x1 * exp(x2 * x3).



Using *Fairmat* you can then try your forward formula using the Fairmat Console tab in which you can type arbitrary expressions using the objects you have created.



2 How to calculate the spot price given the forward price?

In many cases it is necessary to calculate the actual value of a payment function of the underlying for a future date.

In order to calculate it, the inverse function must be calculated:

$$S_t = F_t \cdot e^{-Rf_t(T-t)}$$

Fairmat does this it automatically for you.

Let's suppose that in a future date (01/06/2012) there will be a payment of \$100 and that the Risk free rate is 4.5%. What was the spot price S?

In order to answer that question:

• open Option Map and create a payment block by choosing the payment type, in our case a Custom Option (the pink rumble).





| Option Map | Edit Option | |
|-------------|--|-------------|
| Zoom 100% - | General Timing and Payoff Custom Discounting | |
| | American timing type Continuous Continuous | |
| | Start 0 End (Maturty) 01/06/2012 Relative to exercise | e of option |
| | Payoff expression | |
| | 100 | |
| Map Timing | | |
| | | |
| | Ok | Cancel |

• Open the *custom option* and write maturity and payoff:

• Click the Discounting button and insert the risk free discount rate.

| Structure Data Sources Info | Discounting Settings | |
|---|---|-----------------------------|
| Parameters & Functions Stochastic Processes | Discounting Model Risk Free Rate (RF=) | Use Constant Risk Free Rate |
| Discounting | Only for Constant Risk Free Rate | |
| Option Map | Range | 0 |
| Manage Scenarios | Export | |
| Random Variables | Sensitivity | |
| | | Ok Cancel |

In the discount settings it possible to choose from the following discount models:

- Risk free Rate: if you want o to model a constant risk free rate;
- Deterministic Discounting Expression: if your discounting model depends on a time dependent expression;
- Dynamic Term Structure: if your discounting model depends on a stochastic process;

| Discounting Settings | | × |
|----------------------------------|---|---|
| Discounting Model | Use Constant Risk Free Rate | • |
| Risk Free Rate (RF=) | Use Constant Risk Free Rate Use Deterministic Discounting Expression | |
| Only for Constant Risk Free Rate | Use Dynamic Term Structure | |



- From the Main Menu select the cases:
 - Simulation Date: choose the date where you want evaluate the payoff;

To start the evaluation click on Run Analysis. After that you should see the valuation result in the bottom panel (Valuation tab). You can change the evaluation date and calculate the payoff in other cases.

| File Edit Analysis Settings Tools | Help | | | | | | | | | | |
|-----------------------------------|--------------|----------------------------------|-----------|----------------|-----------|----------|-------------|------------------|------------------------|-----|-------|
| i 🗋 💕 🔒 🚳 i i 🔶 🔶 ≪ i 🗵 | < 🕆 F | FO | | | | | | | | | |
| Current scenario Base case - A | nalysis Valu | uation | - Star | ting node | | • Simu | lation Date | 14/02/2012 📴 Run | Analysis | | |
| Structure Data Sources Info | Option Ma | p | | | | | | febbraio | 2012 | 1 | x G - |
| Parameters & Functions | Zoom 100 | 0% 🔻 | | | | | | lun mar mer gio | ven sab d | lom | |
| Stochastic Processes | | | | | | | | 6 7 8 9 | 10 11 | 12 | |
| Discounting | | | | | | | | 20 21 22 23 | 24 25 | 26 | |
| Option Map | | | | | | | | 5 6 7 8 | 9 10 | 11 | |
| Manage Scenarios | | | | | | | L | | 25/11/2011 | | |
| Random Variables | | | | | | | | | | | |
| | | | | \sim | | | | | | | |
| | | | | | | | | | | | |
| | | | | | | | | | | | |
| | - | | | _ | | | | | | | |
| | Figure # | Node | Туре | Exp. Value | Std. Dev. | Std.Err. | Tech | Time | Scenario | | |
| | 6 | Project's root | Valuation | 98,6809 | 0,0000 | 0,0000 | Simulation | 00:00:00.0200012 | Base case | | |
| | 5 | Project's root | Valuation | 99,8771 | 0,0000 | 0,0000 | Simulation | 00:00:00.0210012 | Base case | | |
| | 4 | Project's root | Valuation | 0,0000 | 0.0000 | 0.0000 | Simulation | 00:00:00.0130008 | Base case | | |
| | 3 | Project's root | valuation | 99,2894 | 0,0000 | 0,0000 | Simulation | 00:00:00.0220013 | base case | | |
| | 1 | Project's root Project's root | Valuation | 98,5718 | 0,0000 | 0,0000 | Simulation | 00:00:00.0210012 | Base case Base case | | |
| | Valuations | Errors Paramete | rs Log F | airmat Console | | | | | | | |

3 Forward Prices with dividends

Let's consider the forward price with periodic interest payments. The forward price F is given by the following expression:

$$F_t = (S_t - I)e^{Rf_t(T-t)}$$

where

$$I = div \cdot e^{-rf(T_1 - t)} + div \cdot e^{-rf(T_2 - t)}) \dots = d1 + d2$$
(1)

with $T_1, T_2...$ time of dividends' payment (indicated with div). The data available are:

- the dividend (*Dividend*);
- the free risk rate (Rf_t) ;
- the first payment date (Ex1);
- the second payment date (Ex2);



• the maturity date (*Maturity*);

| Parameters & Fu | inctions | | | |
|-----------------|---|-------------|---|--|
| Add | Name | Description | Expression | Туре |
| Remove Up Dn | Dividend S Ex1 Ex2 Rf Maturity | | 3 50 01/02/2012 01/06/2012 8% 01/07/2012 | Constant Constant Constant Constant Constant Constant |
| | ٠ | | " | E F |

In *Fairmat* we can resolve it in different ways.

• <u>Method 1:</u> using the Sum Operator



In this case the option map's blocks are:

- blocks d1 and d2: calculate the dividend's and discount them at the valuation date. Double click on it and write the dividend value, the date of payment (End (Maturity)) and the discount in Discouting section:

| Edit Option \cdots 🕘 🚾 | Edit Option | |
|--|--|--------|
| General Timing and Payoff Custom Discounting | General Tining and Payoff Custom Discounting | |
| American liming type Continuous | Custon Discounting Use a custon discounting structure | |
| Start 0 Relative Time | Decounting Model Use Constant Risk Free Rate | |
| End (Maturity) EX1 | Risk Free Rate (RF+) | |
| ia +e, Description Homula | Only for Constant Risk Free Rate Range 0 | |
| Payoff expression | E Beot | |
| Dividend | E Senativity | |
| | impact in the second se | |
| | | |
| | | |
| Ok Cancel | | Cancel |

You can then sum the discounted values of d1 and d2 by using Sum Operator, hence calculating then term I of formula (1).



- Block *Capit I* capitalizes dividend *I*:

| | | | | | | (mar.) |
|---|------------------------------------|-----------------|------------------|----------|------|---------|
| unctional Operat | or | | | | | |
| Description Ca | et l | | | | ld | 172 |
| Name | | ld | | | | |
| Behavior | Calculates | the transformat | ion at a given o | decision | date | - |
| Decision/Value d | ste | | 0 | | | |
| Analytic Expressio | ny = F(X1,,) | (n) | | | | |
| x1*exp(Rf*M | aturity) | | | | | |
| x1*exp(Rf*M | aturity) bles | | | | | |
| x1*exp(Rf*M Independent Varia Option | aturity) bles Variable | | | | | |
| x1*exp(Rf*M Independent Varia Option Sum dividends | aturity) bles Variable ×1 | | | ОК | | Cancel |

note: in the bottom of the function **operator**'s window there is variable x1. This indicates the values of subsequent nodes in the option map: in our case the dividend's sum I.

- Block Capital S capitalizes the underlying price (S_t) (as before):

| Several Timing an | d Payoff L Custors D | howsten | |
|-----------------------------------|----------------------|------------------------------------|--|
| American timing by | PR Continuous | | |
| Continuous | | | |
| Start | | 0 Relative Time | |
| End (Maturity) | | 0 id:+6, Description:+formula * | |
| | | | |
| | | | |
| David excession | | | |
| Payoff expression | 1 - 21.2 | | |
| Payoff expression S*exp(Rf*Mai | turity) | | |
| Payoff expression S*exp(Rf*Mai | turity) | | |
| Payoff expression S*exp(Rf*Ma | turity) | | |
| Payoff expression S*exp(Rf*Ma | turity) | | |
| Payoff expression S*exp(Rf*Mai | turity) | | |
| Payoff expression S*exp(Rf*Ma | turity) | | |
| Payoff expression S*exp(Rf*Ma | turity) | | |

 Block *formula* subtracts the underlying price and dividends (using the names of variables show in the form)

| Functional Op | perator | | | | | | | × |
|--------------------------------|-----------------------|-------------------|-------------|-------------|-----------|----------|----|------|
| Description | fomula | | | | | ld | | 6 |
| Name | | | ld | | | | | |
| Behavior | | Calculates the tr | ansformatio | on at a gir | ven decis | ion date | | • |
| Decision/Val Analytic Expre | ue date ession y - | = F(X1, , Xn) | | 0 | | | | |
| (x2-x1) | Variables | 3 | | | | | | |
| Option | | Variable | | | | | | |
| Capit I Capitalize S | | x1 x2 | | | | | | |
| | | | | | 0 | K | Ca | ncel |

In order to get the value indicate the Start date in the Simulation Date field and the starting node and click on Run.



An alternatively you can select the starting node or a particular node, for example blocks d1 or d2, by right clicking.

| File Edit Analysis Settings Tools | Help | | | | | | | | | | | | | |
|-----------------------------------|---------------|--------------------|-------------------|---------------|-----------|---------------------------|------------|------------|-----------|-----------------|-------------|-------------------------|----------------------|----------------|
| i 🗅 💕 🔜 🍕 i 🔶 🔶 🔍 🚺 | Σ \prec 😤 🗉 | FØ | | | | | | | | | | | | |
| Current scenario Base case 🔹 | Analysis Valu | ation | - Star | rting node | | Simul | ation Date | 01/01/2012 | 💷 🛛 Run | Analys | is | | | |
| Structure Data Sources Info | Option Ma | р | | | | | | | | | | | - |) 🗗 🛛 🔀 |
| Parameters & Functions | Zoom 100 | 1% 🔻 | | | | | | | | | | | | |
| Stochastic Processes | | | | | | | | | | | | | | |
| Discounting | | | | | | \frown | | Analysis |) | | Dynamic | and Volatility Analysis | CTRL+MAIUSC+D | |
| Option Map | | | | F(*) | Sum | m dividondi | | Properties | | | Impact A | nalysis | | |
| Manage Scenarios | | F(*) | | Capit I | 00 | Inpividend | <u> </u> | Delete | | | Risk Anal | /sis | CTRL+MAIUSC+R | |
| | | | ömula | | | | | Cut | CTRL+X | | Scenario | (What-if) Analysis | | |
| Handom Vanables | | | $\langle \rangle$ | | | | | Сору | CTRL+C | | Sensitivity | / Analysis | | • |
| | | | | | | | _ | | | | Valuation | | CTRL+MAIUSC+V | |
| | | | | Capitali | ze S | | | | | _ | | Calculates the mark | to market (or value) | of the project |
| | | | 1 | | | | | | | | | | | |
| | Figure # | Node | lype | Exp. Value | Std. Dev. | Std.Err. | lech | lime | | Scena | no | | | |
| | 3 | Project's root | Valuation | 45,9088 | 0,0000 | 0,0000 | Simulatio | n 00:00:00 | 0.1770101 | Base | ase | | | |
| | 2 | d2 | Valuation | 2,9020 | 0,0000 | 0,0000 | Simulatio | n 00:00:00 | 0.1820104 | Base of Page of | ase | | | |
| | | ui | Valuation | 2,3737 | 0.0000 | 0,0000 | Sinulatio | 00.00.00 | 0.2740107 | Dase | | | | |
| | | | | | | | | | | | | | | |
| | | | | | | | | | | _ | | | | |
| l | Valuations | Errors Parameter | s Log F | aimat Console | | | | | | | | | | |

• <u>Method 2</u>: You can omit the Sum Operator by composing the *d1* and *d2* blocks as in figure.



• <u>Method 3</u>: use vectors.

In *Fairmat* you can use vectors (Parameters & Functions \rightarrow Vector of values/expressions/dates) to model sequences of payments.



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3 Forward Prices with dividends



Editing Vectors: in the Expression column you can write the payment dates and in the Value columns you can see the relative value from the valuation date.

In Option Map use an Option Strip block in order to actualize dividends in one step:



Options Strips simplif the repetition of similar payoffs and exercise dates (parametrized by the character **#**) by summing them over the components of an input vector³.

 $^{^3\}mathrm{If}$ you refer to a vector you must write the symbol @ before vector name.



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Here you can use Discounting for actualize payoff:

| Parametric Option Strip | Parametric Option Strip |
|--|--|
| General Timing and payoff Custom Discounting | General Timing and payoff Custom Discounting |
| From Step To 1 1 length(@Ex) View Parametric exercise date (#) | Custom Discounting Use a custom discounting structure |
| Ex[#] | Discounting Model Use Constant Risk Free Rate |
| Parametric payoff (#) Dividend*exp(Rf*Maturity) | Risk Free Rate (RF=) Rf Only for Constant Risk Free Rate Range 0 |
| Callability No callability Parametric alternative payoff (#) | Export Sensitivity Impact |
| | Ok Cancel |

Parametric Options Strip the payoff $Dividend * exp * (Rf_t * Maturity)$ is calculated for every payment date defined by the expression Ex(#), where # takes the values 1,2,...,(@Ex).

Now you can calculate the forward price, as in previous pages examples.



4 How to calculate the value of a simple Forward Contract

The value of a forward contract at maturity depends on the relationship between the delivery price (K), which is settled in advance, and the underlying price (S) at the delivery date. Consider the following example where the actual value f of a forward contract can be calculate by the following expressions:

- long position: $f = (F_t K)e^{-Rf_t(T-t)};$
- short position: $f = (K F_t)e^{-Rf_t(T-t)}$



maturity of the following variables:

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Parameters & Functions Add Name Description Expression Туре Dividend Constant 3 Remove 50 Constant S Rf 8% Constant Up Dn 40 Constant Κ Maturity 01/07/2012 Constant Ex Vector Long_Short 1 Constant ٠

in order to calculate the value of the contract in any date before the

Create the contract structure using the Option map. The red circle refers to the previous example.



The block *forward price* computes the forward price value:

| Description | forward price Id 841 |
|-----------------------------------|--|
| Behavior | Calculates the transformation at a given decision date |
| Decision/Value Analytic Expres | date 0 |
| Long_Short | *(x1-K*exp(-Rf*Maturity)) |
| Option | Variable Variable |
| formula | x1 |
| | |
| | |
| | OK Cancel |



4 How to calculate the value of a simple Forward Contract



You can then evaluate the other components:

| Figure # | Node | Туре | Exp. Value | Std. Dev. | Std.Err. | Tech | Time | Scenario | | |
|--|---------------------------------|-----------|------------|-----------|----------|------------|------------------|-----------|--|--|
| 4 | vector_dividend | Valuation | 6,1208 | 0,0000 | 0,0000 | Simulation | 00:00:00.2680153 | Base case | | |
| 3 | Capitalize Underlying Price (S) | Valuation | 52,0292 | 0,0000 | 0,0000 | Simulation | 00:00:00.2330133 | Base case | | |
| 2 | fomula | | 45,9084 | 0,0000 | 0,0000 | Simulation | 00:00:00.3160181 | Base case | | |
| 1 Project's root | | Valuation | 7,4684 | 0,0000 | 0,0000 | Simulation | 00:00:00.6250357 | Base case | | |
| Valuations Errors Parameters Log Fairmat Console | | | | | | | | | | |

