Tutorial | fairmat

Forward Prices & Forward Contracts

In this tutorial we show you how to use ***Fairmat Academic*** to solve exercises on forward prices modelling, by showing you how to answer questions found in *John C. Hull Options, futures and other derivatives [Chapter 3 "Determination of forward and futures prices" , 5^{th} Edition].*

You can also find a video for this tutorial at: http://youtu.be/Bq2B61cf9FU

1 Zero coupon bond

We want to calculate a forward price, at time t[1] of an underlying price that does not make periodic interest payments:

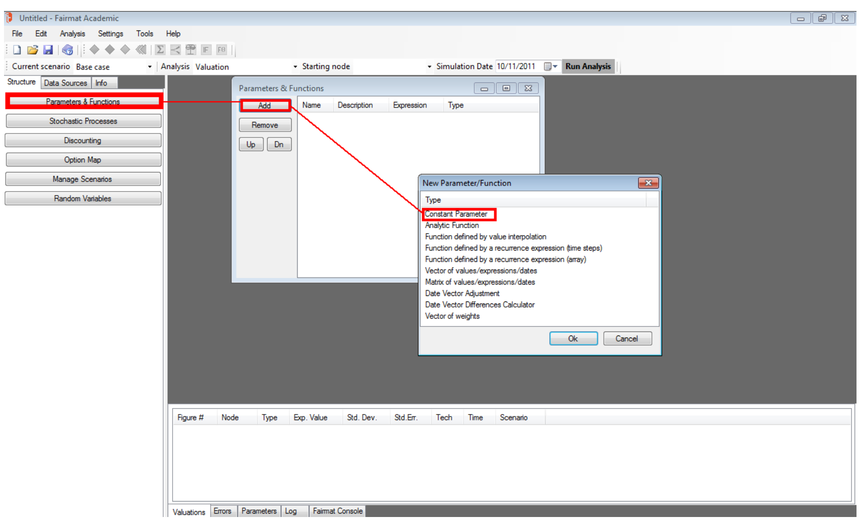
$$F_t = S_t \cdot e^{Rf_t(T-t)}$$

The data available are:

- underlying price (**S**), price in stipulation date;
- Risk Free (**Rf**);
- Maturity (**EX**)[2];

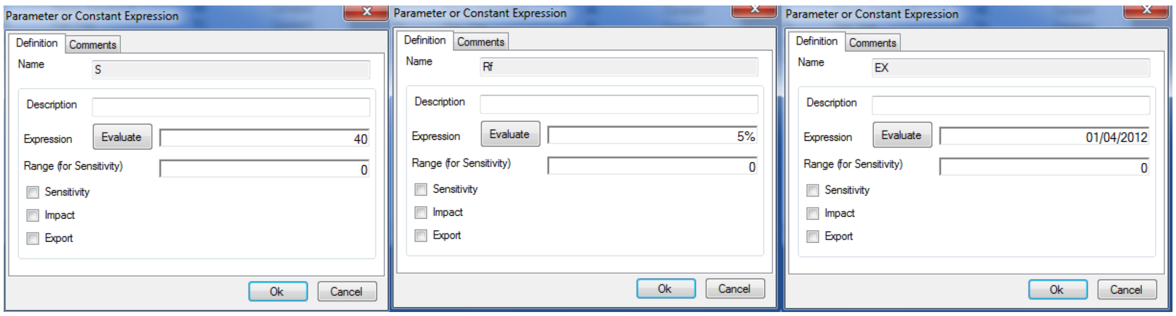
With Fairmat, the first step is creating the problem constants as follows:

1. Click in the window *Structure* the `Parameters & Functions`;
2. Click in the new opened window the `Add` button ;
3. Select `Constant Parameter` on the list;



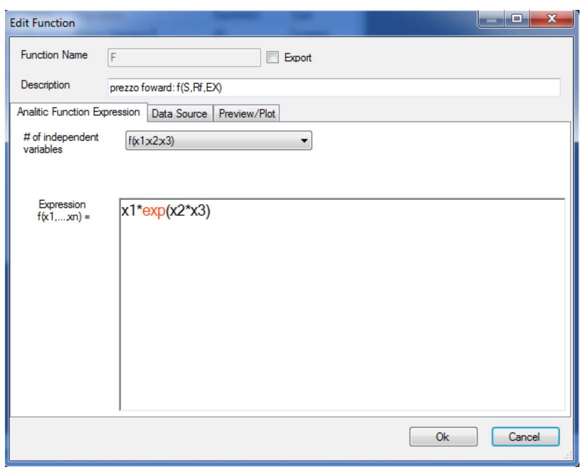
[1] t is indicated in *Simulation Dates*.
[2] EX is T in the formula.

In order to focus on a specific case, constants can be initialized as follows: $S_t = 40\$$, Rf_t=5%, EX=01/04/2012



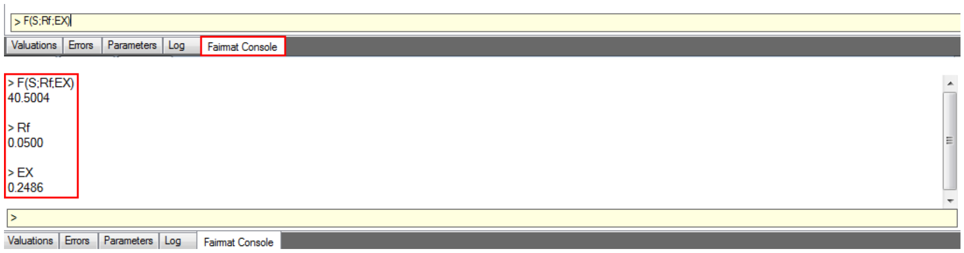
Then we can create the forward expression using Fairmat's analytic functions (just go to `Parameters & Functions` and select `Analytic Function`).

In this example we model forward prices as a function of three independent variables, which as indicated 1=S_t x2=Rf_t x3=Maturity:



In the expression textbox just write the forward formula $x1 * exp(x2 * x3)$.

Using ***Fairmat*** you can then try your forward formula using the `Fairmat Console tab` in which you can type arbitrary expressions using the objects you have created.



2 How to calculate the spot price given the forward price?

In many cases it is necessary to calculate the actual value of a payment function of the underlying for a future date.

In order to calculate it, the inverse function must be calculated:

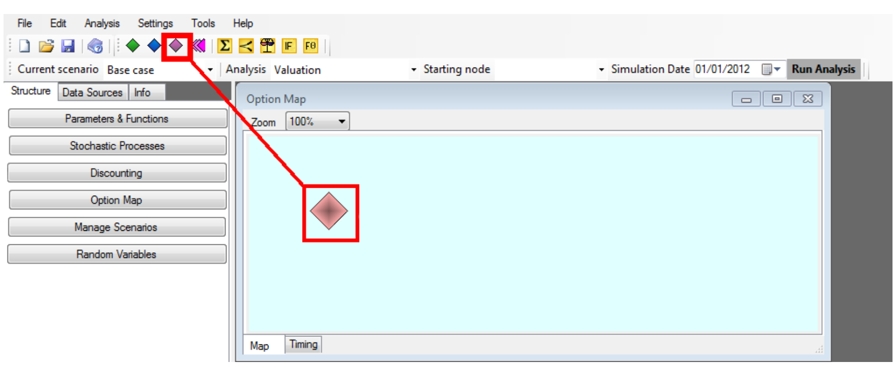
$$S_t = F_t \cdot e^{-Rf_t(T-t)}$$

Fairmat does this it automatically for you.

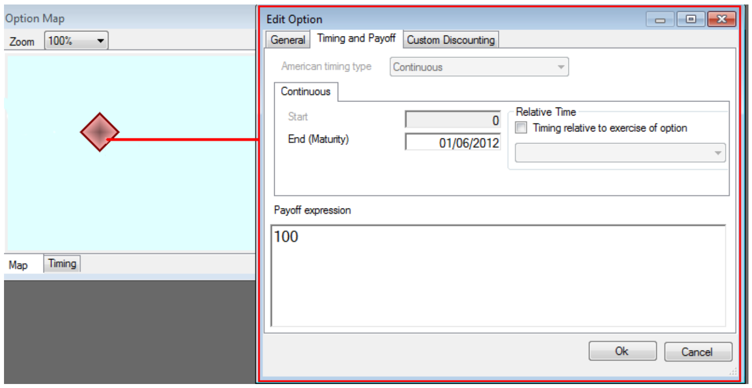
Let's suppose that in a future date (01/06/2012) there will be a payment of \$100 and that the Risk free rate is 4.5%. What was the spot price S?

In order to answer that question:

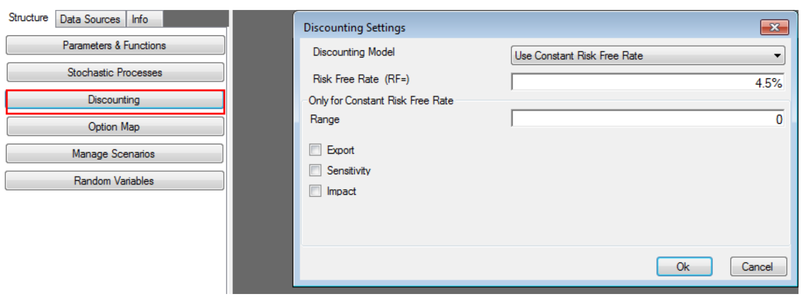
- open `Option Map` and create a payment block by choosing the payment type, in our case a Custom Option (the pink rumble) .



- Open the *custom option* and write maturity and payoff:

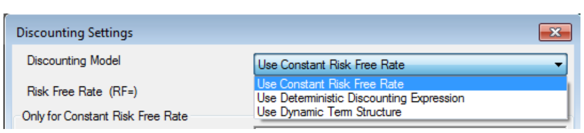


- Click the `Discounting` button and insert the risk free discount rate.



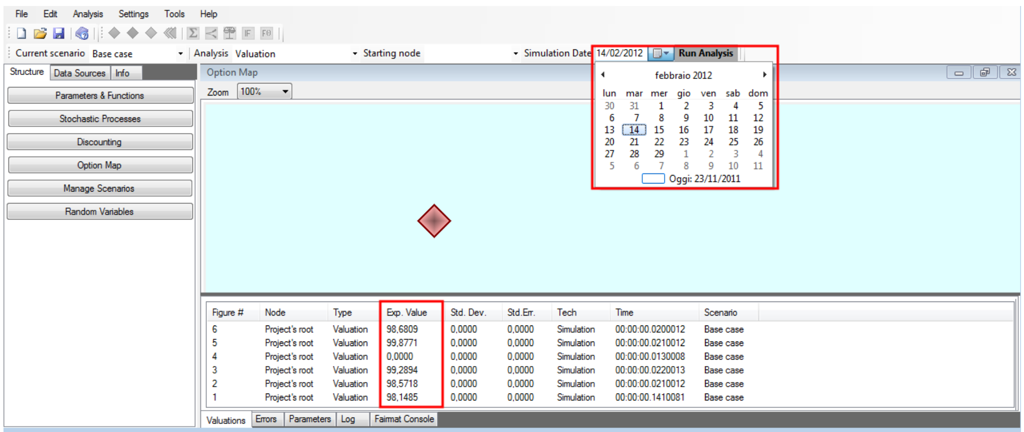
In the discount settings it possible to choose from the following discount models:

- Risk free Rate: if you want o to model a constant risk free rate;
- Deterministic Discounting Expression: if your discounting model depends on a time dependent expression;
- Dynamic Term Structure: if your discounting model depends on a stochastic process;



- From the `Main Menu` select the cases:
  - Simulation Date: choose the date where you want evaluate the payoff;

  To start the evaluation click on Run Analysis. After that you should see the valuation result in the bottom panel (Valuation tab). You can change the evaluation date and calculate the payoff in other cases.



3 Forward Prices with dividends

Let's consider the forward price with periodic interest payments. The forward price F is given by the following expression:

$$F_t = (S_t - I)e^{Rf_t(T-t)}$$

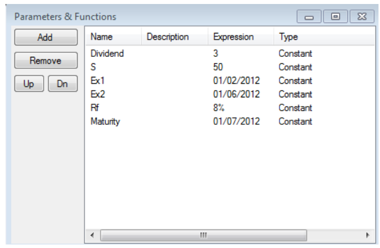
where

$$I = div \cdot e^{-rf(T_1-t)} + div \cdot e^{-rf(T_2-t)})... = d1 + d2 \tag{1}$$

with $T_1, T_2...$ time of dividends' payment (indicated with div).
The data available are:

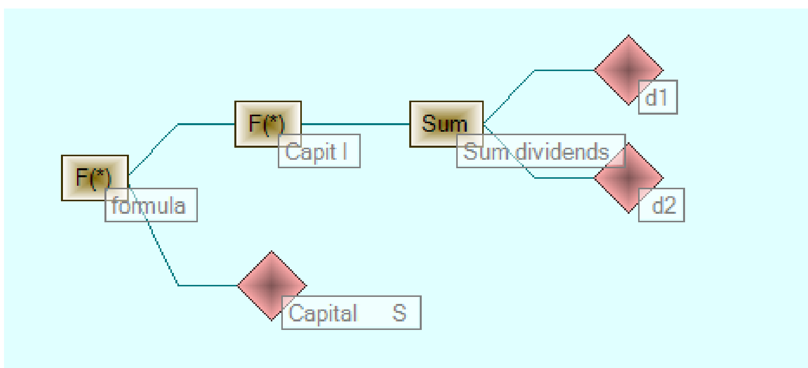
- the dividend (*Dividend*);
- the free risk rate (Rf_t);
- the first payment date (*Ex1*);
- the second payment date (*Ex2*);

- the maturity date (*Maturity*);



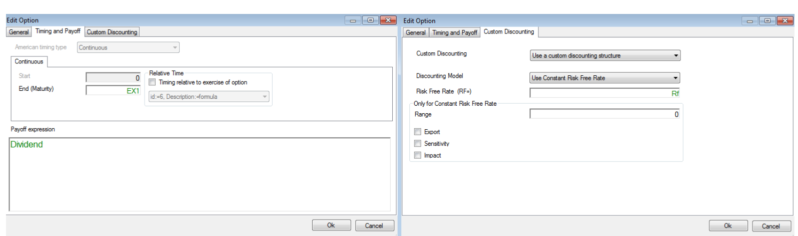
In ***Fairmat*** we can resolve it in different ways.

- Method 1: using the `Sum Operator`



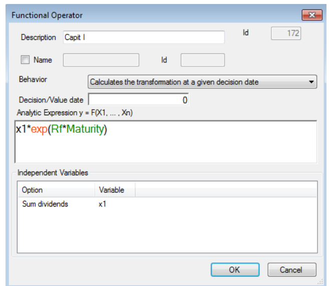
In this case the option map's blocks are:

- blocks *d1* and *d2*: calculate the dividend's and discount them at the valuation date. Double click on it and write the dividend value, the date of payment (End (Maturity)) and the discount in `Discouting` section:



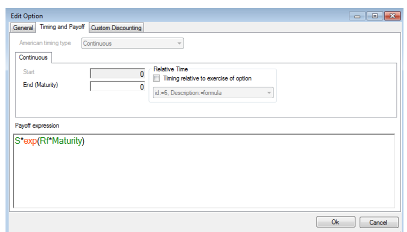
You can then sum the discounted values of *d1* and *d2* by using `Sum Operator`, hence calculating then term I of formula (1).

– Block *Capit I* capitalizes dividend I:

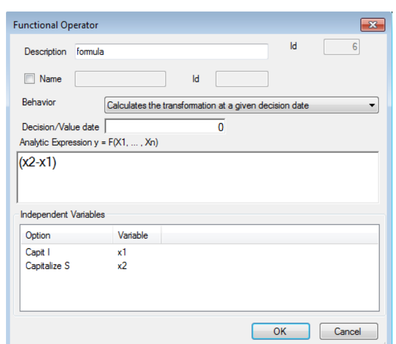


note: in the bottom of the function `operator`'s window there is variable *x1*. This indicates the values of subsequent nodes in the option map: in our case the dividend's sum *I*.

– Block *Capital S* capitalizes the underlying price (S_t) (as before):

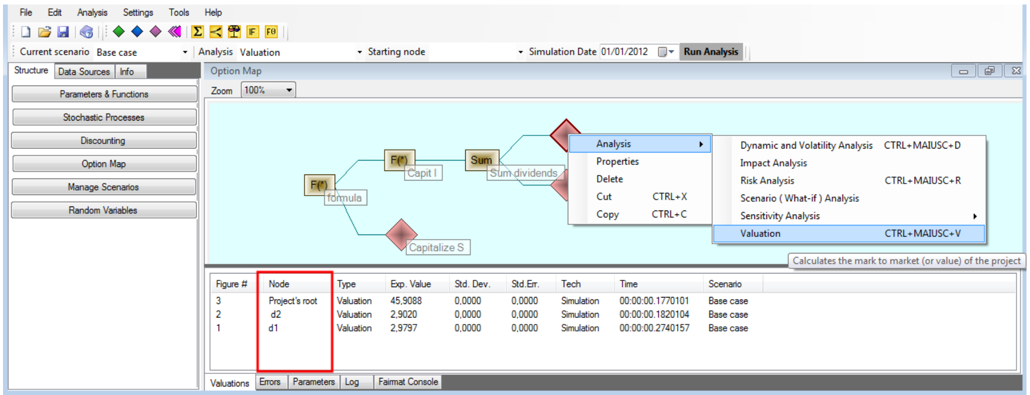


– Block *formula* subtracts the underlying price and dividends (using the names of variables show in the form)

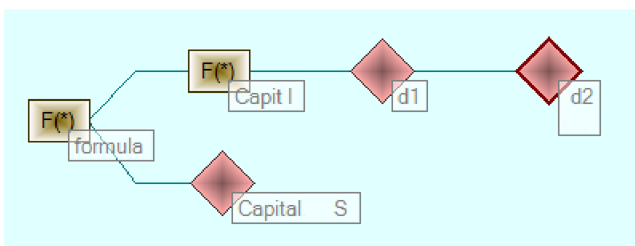


In order to get the value indicate the Start date in the `Simulation Date` field and the starting node and click on `Run`.

An alternatively you can select the starting node or a particular node, for example blocks *d1* or *d2*, by right clicking.

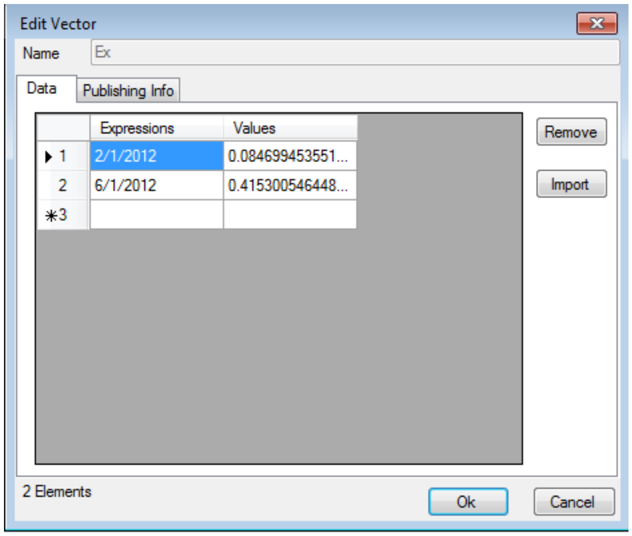


- Method 2: You can omit the `Sum Operator` by composing the *d1* and *d2* blocks as in figure.



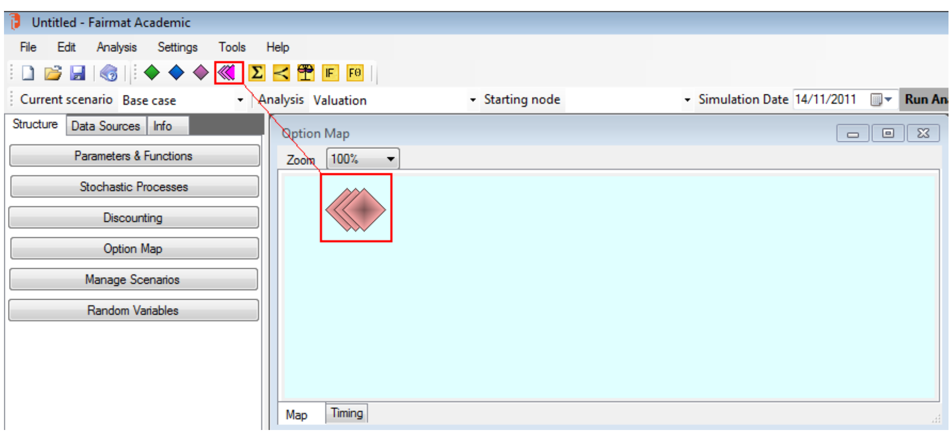
- Method 3: use vectors.

  In ***Fairmat*** you can use vectors (`Parameters & Functions` → `Vector of values/expressions/dates`) to model sequences of payments.



Editing Vectors: in the `Expression` column you can write the payment dates and in the `Value` columns you can see the relative value from the valuation date.

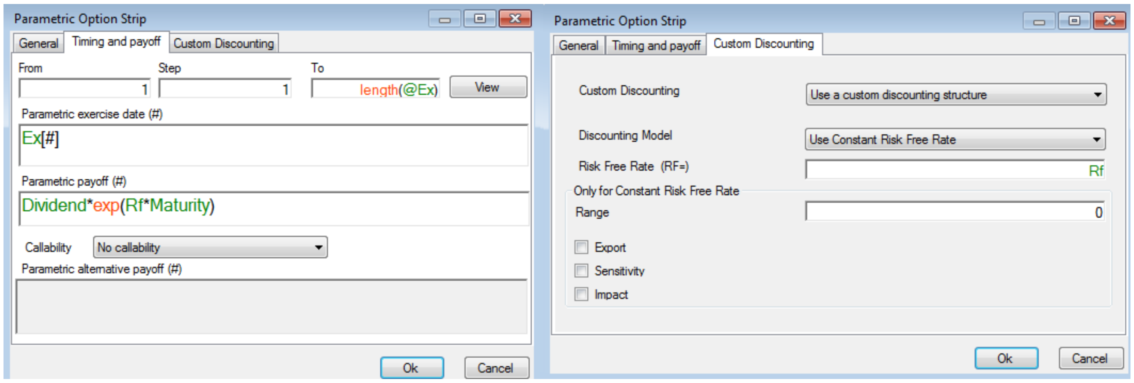
In `Option Map` use an `Option Strip` block in order to actualize dividends in one step:



`Options Strips` simplif the repetition of similar payoffs and exercise dates (parametrized by the character `#`) by summing them over the components of an input vector[3].

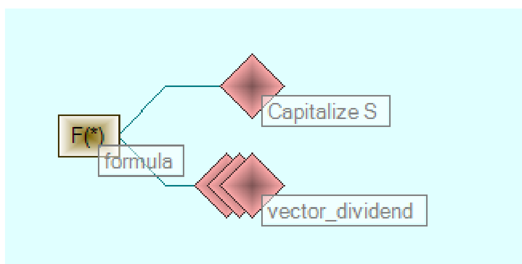
[3] If you refer to a vector you must write the symbol @ before vector name.

Here you can use Discounting for actualize payoff:



`Parametric Options Strip` the payoff $Dividend * exp * (Rf_t * Maturity)$ is calculated for every payment date defined by the expression *Ex(#), where # takes the values 1,2,...,(@Ex).*

Now you can calculate the forward price, as in previous pages examples.



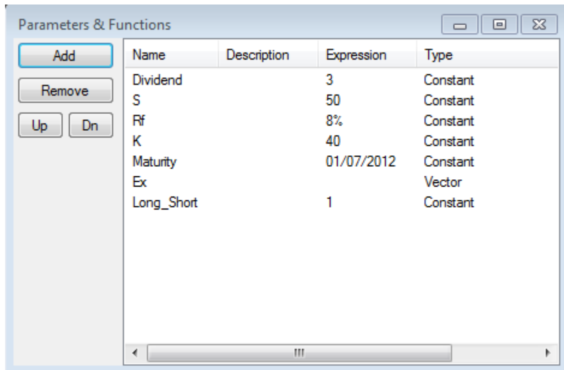
4 How to calculate the value of a simple Forward Contract

The value of a forward contract at maturity depends on the relationship between the delivery price (K), which is settled in advance, and the underlying price (S) at the delivery date. Consider the following example where the actual value f of a forward contract can be calculate by the following expressions:

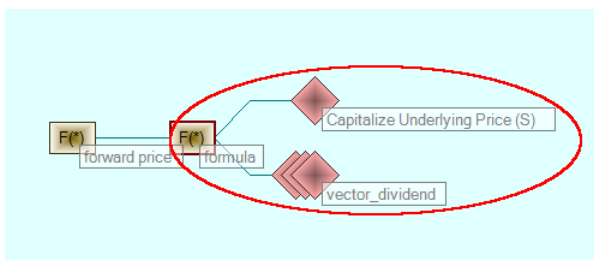
– long position: $f = (F_t - K)e^{-Rf_t(T-t)}$;

– short position: $f = (K - F_t)e^{-Rf_t(T-t)}$

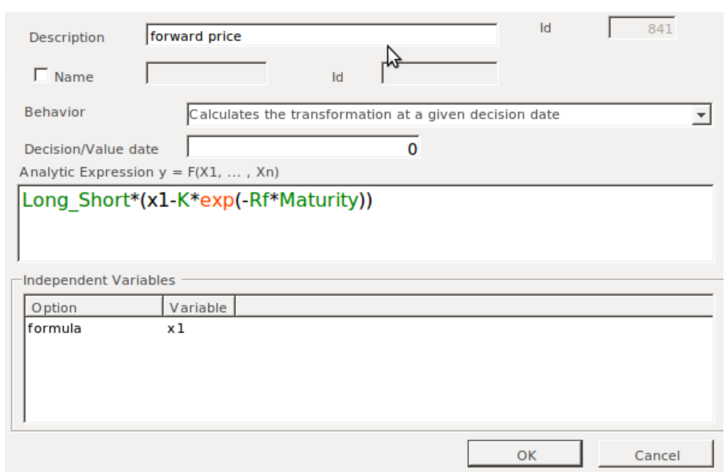
in order to calculate the value of the contract in any date before the maturity of the following variables:



Create the contract structure using the `Option map`. The red circle refers to the previous example.



The block *forward price* computes the forward price value:



You can then evaluate the other components:

Figure #	Node	Type	Exp. Value	Std. Dev.	Std.Err.	Tech	Time	Scenario
4	vector_dividend	Valuation	6,1208	0,0000	0,0000	Simulation	00:00:00.2680153	Base case
3	Capitalize Underlying Price (S)	Valuation	52,0292	0,0000	0,0000	Simulation	00:00:00.2330133	Base case
2	formula	Valuation	45,9084	0,0000	0,0000	Simulation	00:00:00.3160181	Base case
1	Project's root	Valuation	7,4684	0,0000	0,0000	Simulation	00:00:00.6250357	Base case

Valuations | Errors | Parameters | Log | Fairmat Console